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Complex Analysis.

Today's IIScian Approach

Interview Prep^n for PhD:-

Let f be meromorphic function on $\mathbb{C}$ such that $|f(z)| \geq |z|$, at each z where f is holomorphic. Then

(a) No such f exists

(b) Such an f is entire

(c) There is a unique f satisfying the given conditions

(d) There is an $A \in \mathbb{C}$ with $|A| \geq 1$ s.t. $f(z) = Az \quad \forall z \in \mathbb{C}$.

Sol^n:- How to THINK

First recall some basic facts!

(*) • If f and g are entire function and $|f| \leq |g|$ then $f = \alpha g$ for some $\alpha \in \mathbb{C}$ (Justify)

• If f is meromorphic on $\mathbb{C} \Rightarrow f = \frac{g}{h}$ where g and h are holomorphic on $\mathbb{C}$

• Thus $\left|\frac{g}{h}(z)\right| \geq |z| \Rightarrow |h(z)\, z| \leq |g(z)|$

• Hence apply (*) (b) and (d) are write option.