

Today's IIScian Approach

Q Suppose ϕ is an isomorphism from $\mathbb{Z}_3 \times \mathbb{Z}_5$ to \mathbb{Z}_{15} and $\phi(2,3) = 2$.

Then find $(a,b) \in \mathbb{Z}_3 \times \mathbb{Z}_5$ such that $\phi(a,b) = 1$.

THINK

Solⁿ :-

How to

Recall some basic facts:

- If $\phi: G_1 \rightarrow G_2$ is an isomorphism then
 - $\phi(a) = o(a)$, $\forall a \in G_1$
 - $\phi(a^k) = (\phi(a))^k$, $\forall k \in \mathbb{Z}$
 - $G_1 = \langle a \rangle \Leftrightarrow G_2 = \langle \phi(a) \rangle$

• Here $\langle (2,3) \rangle = \mathbb{Z}_3 \times \mathbb{Z}_5$ and $\langle 2 \rangle = \mathbb{Z}_{15}$

• In \mathbb{Z}_{15} , $2^8 = 1$ (How?? Justify)

• $\therefore \phi(2,3) = 2 \Rightarrow \phi((2,3)^8) = 2^8$

$\Rightarrow \phi(2^8, 3^8) = 1 \Rightarrow \phi(1, 4) = 1$

$2^8 = 1$ in \mathbb{Z}_3

$3^8 = 4$ in \mathbb{Z}_5 .

Hence $(a,b) = (1,4)$

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