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Today's IIScian Approach

Q Let $f : (0,\infty) \to \mathbb{R}$ be a differentiable function s.t.

$$\lim_{x\to+\infty} [f(x) + f'(x)] = 0$$

Then $\lim_{x\to+\infty} f(x) = 0$. (True | false)

Sol:- How to $\boxed{\text{THINK}}$

First recall some basic facts:-

- If g and h are diff. on on $[a,b]$ then $\exists$ $c \in (a,b)$ s.t. $\dfrac{g(b)-g(a)}{f(b)-f(a)} = \dfrac{g'(c)}{f'(c)}$ [Prove this]
- Let $\epsilon > 0$, then $\exists$ $M > 0$ s.t. $|f(x) + f'(x)| < \epsilon$ for $x \geq M$. (WHY)
- Take $g(x) = e^{x} f(x)$, $h(x) = e^{x}$, then $\exists$ $c \in (M,x)$ s.t. $\dfrac{g(x)-g(M)}{f(x)-f(M)} = \dfrac{g'(c)}{f'(c)} \Rightarrow \dfrac{e^{x}f(x) - e^{M}f(M)}{e^{x} - e^{M}} = f(c) + f'(c)$.
- After simplification $|f(x)| < |f(M)| e^{M-x} + \epsilon |1 - e^{M-x}|$

Now take $x \to \infty$, $|f(x)| \to 0$ [Justify])

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