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1 May Crash Course
NET JUNE

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Today's IIScian Approach

(Q) Let R be a finite non-zero commutative ring with Unity. Then which of the following statements are necessarily true?

(a) Every prime ideal ideal of R is maximal

(b) any non-zero element of R is either a unit or a zero divisor.

Sol:- How to $\boxed{\text{THINK}}$

Your dedication + my direction ⇒ Result !!!

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Recall some basic facts:-

- Every Finite Integral domain is field.
- Here R is CRU and finite, let $0 \neq a \in R$
- If 'a' is zero divisor then nothing to show
- So let 'a' is NOT a zero divisor.
- consider $\{aa_1, aa_2, \dots, aa_n\}$ where $R = \{a_1, \dots, a_n\}$
- If $aa_i = aa_j \Rightarrow a(a_i - a_j) = 0 \Rightarrow a_i = a_j$ →←
- Thus $\exists\ i$ s.t. $aa_i = 1$ (WHY ??), so 'a' is Unit
- Next let I be a prime ideal in R. Then $\frac{R}{I}$ is finite I.D ⇒ Field ⇒ I is maximal