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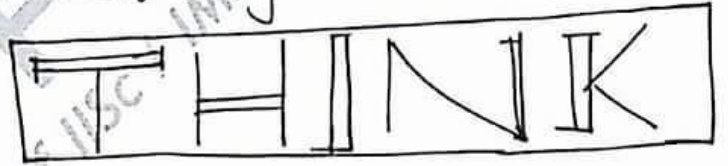
Today's IIScian Approach

Let $f_n: [0,1] \rightarrow \mathbb{R}$ be given by $f_n(x) = \frac{2x^2}{x^2 + (1-2nx)^2}$
 $n = 1, 2, \dots$. Then the sequence $\{f_n\}$

- ① converges point wise but NOT uniformly on $[0,1]$
- ② does NOT converge uniformly on $[0,1]$ but has a subsequence that converges uniformly on $[0,1]$.
- ③ does NOT converge uniformly ④ converges uniformly.

Qn :-

How to



Hint recall some basic facts:-

- $f_n \rightarrow f$ unif on E $\Leftrightarrow \|f_n - f\|_\infty \rightarrow 0$
where $\|f\|_\infty = \sup_{x \in E} |f(x)|$.
- clearly $\lim_{n \rightarrow \infty} f_n(x) = 0 \quad \forall x \in [0,1]$, (Just by)
- But $\|f_n - 0\|_\infty = \|f_n\|_\infty = 2, \quad \forall n \in \mathbb{N}$. [WHY??].
- Thus $f_n \not\rightarrow 0$ uniformly on $[0,1]$.
- In fact for any subseq $\{f_{n_k}\}, \|f_{n_k} - 0\|_\infty = 2$.
∴ hence $f_{n_k} \not\rightarrow 0$ uniformly.

NET December 2017 ... from ... 3rd June ... Get your concepts Here ↴