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Pt. + Unif → converg

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Today's IIScian Approach

Let $f_n : [0,1] \to \mathbb{R}$ be given by $f_n(x) = \dfrac{2x^2}{x^2+(1-2nx)^2}$, $n = 1, 2, \ldots$. Then the sequence $\{f_n\}$

1. converges point wise but NOT uniformly on [0,1]
2. does NOT converge uniformly on [0,1] but has a subsequence that converges uniformly on [0,1].
3. does NOT converge uniformly
4. converges uniformly.

Sol :- How to $\boxed{\text{THINK}}$

First recall some basic facts :-

- $f_n \to f$ unif on $E \iff \|f_n - f\|_\infty \to 0$ where $\|f\|_\infty = \sup_{x\in E} |f(x)|$.
- clearly $\lim_{n\to\infty} f_n(x) = 0 \quad \forall x \in [0,1]$. (Justify)
- But $\|f_n - 0\|_\infty = \|f_n\|_\infty = 2. \quad \forall n \in \mathbb{N}$. [WHY??].
- Thus $f_n \not\to 0$ uniformly on [0,1].
- In fact for any subseq $\{f_{n_k}\}$, $\|f_{n_k} - 0\|_\infty = 2$. & hence $f_{n_k} \not\to 0$ uniformly.