

Today's IIScian Question

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Let $A := (a_{ij})$ be a 100×100 matrix

given by $a_{ij} = i^2 + j^2$
 $1 \leq i, j \leq 100.$

Then the rank of A ???

Solⁿ 1- If $n \geq 2$ then

$$A = \begin{bmatrix} 1^2+1^2 & 1^2+2^2 & \dots & 1^2+n^2 \\ 2^2+1^2 & 2^2+2^2 & \dots & 2^2+n^2 \\ \vdots & \vdots & \ddots & \vdots \\ n^2+1^2 & n^2+2^2 & \dots & n^2+n^2 \end{bmatrix}_{n \times n}$$

Operate $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_2, \dots, R_n \rightarrow R_n - R_{n-1}$

$$A \sim \begin{bmatrix} 1^2+1^2 & 1^2+2^2 & \dots & 1^2+n^2 \\ 2^2-1^2 & 2^2-1^2 & \dots & 2^2-1^2 \\ 3^2-2^2 & 3^2-2^2 & \dots & 3^2-2^2 \\ \vdots & \vdots & \ddots & \vdots \\ n^2-(n-1)^2 & n^2-(n-1)^2 & \dots & n^2-(n-1)^2 \end{bmatrix}_{n \times n} =$$

$$= \begin{bmatrix} 2 & 5 & \dots & n^2+1 \\ 3 & 3 & \dots & 3 \\ 5 & 5 & \dots & 5 \\ \vdots & \vdots & \ddots & \vdots \\ 2n-1 & 2n-1 & \dots & 2n-1 \end{bmatrix}_{n \times n} \sim \begin{bmatrix} 2 & 5 & \dots & n^2+1 \\ 1 & 1 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

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