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Complex Analysis

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Today's IIScian Approach

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Let $\mathbb{D}$ be the unit disc in $\mathbb{C}$. $g : \mathbb{D} \to \mathbb{D}$ be holomorphic, $g(0) = 0$ and

$$h(z) := \begin{cases} \frac{g(z)}{z} & : z \in \mathbb{D}, z \neq 0 \\ g'(0) & : z = 0 \end{cases}$$

TWOTF is/are true.

① h is holomorphic in $\mathbb{D}$ ④ $|g(\frac{1}{2})| \leq \frac{1}{2}$.

② $h(\mathbb{D}) \subseteq \overline{\mathbb{D}}$ ③ $|g'(0)| > 1$

Sol^n:- How to THINK

First recall some basic facts:-

- $\because$ g is holo on $\mathbb{D}$ $\Rightarrow$ h is analytic on $\mathbb{D}$ as $h(0) = g'(0) = \lim_{z \to 0} \frac{g(z) - g(0)}{z - 0} = \lim_{z \to 0} h(z)$.
- By Maximum modulus principle; $\forall$ $0 < r < 1, |z| \leq r$ we have $|h(z)| = \frac{|g(z)|}{|z|} \leq \frac{1}{r}$ (Justify).
- Now take $r \to 1$ we have $|h(z)| \leq 1$ $\forall$ $z \in \mathbb{D}$ $\Rightarrow$ $h(\mathbb{D}) \subseteq \overline{\mathbb{D}}$.
- Thus $|h(0)| \leq 1 \Rightarrow |g'(0)| \leq 1$
- $|h(z)| \leq 1 \Rightarrow |g(z)| \leq |z| \Rightarrow |g(\frac{1}{2})| \leq \frac{1}{2}$.

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