

Today's IIScian Approach

NET-June

Let \mathbb{D} be the unit disc in \mathbb{C} . $g: \mathbb{D} \rightarrow \mathbb{D}$ be holomorphic, $g(0) = 0$ and

$$h(z) := \begin{cases} \frac{g(z)}{z} & ; z \in \mathbb{D}, z \neq 0 \\ g'(0) & ; z = 0 \end{cases}$$

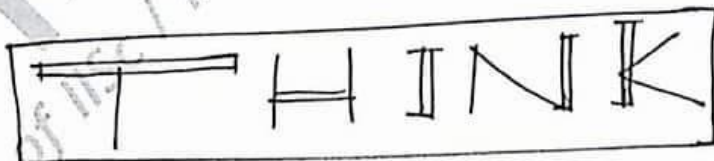
TWOTF is false true.

- ① h is holomorphic in \mathbb{D}
- ② $h(\mathbb{D}) \subseteq \overline{\mathbb{D}}$
- ③ $|g'(0)| > 1$
- ④ $|g(\frac{1}{2})| \leq \frac{1}{2}$

Sol:-

flow

to



First recall some basic facts:-

- $\because g$ is holo on $\mathbb{D} \Rightarrow h$ is analytic on \mathbb{D}
- as $h(0) = g'(0) = \lim_{z \rightarrow 0} \frac{g(z) - g(0)}{z - 0} = \lim_{z \rightarrow 0} h(z)$.
- By Maximum modulus principle; $\forall 0 < r < 1, |z| \leq r$ we have $|h(z)| = \frac{|g(z)|}{|z|} \leq \frac{1}{r}$ (Just by).
- Now take $r \rightarrow 1$ we have $|h(z)| \leq 1$ $\forall z \in \mathbb{D} \Rightarrow h(\mathbb{D}) \subseteq \overline{\mathbb{D}}$.
- Thus $|h(0)| \leq 1 \Rightarrow |g'(0)| \leq 1$
- $|h(z)| \leq 1 \Rightarrow |g(z)| \leq |z| \Rightarrow |g(\frac{1}{2})| \leq \frac{1}{2}$.

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