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Centre for Excellence in MATHEMATICS
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Real Analysis

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Today's IIScian Approach

Q. Let $x_1 = 0,\ x_2 = 1$ and $x_n = \dfrac{x_{n-1} + x_{n-2}}{2}$; $n \geq 3$

Then WOTF is/are true

(a) $\lim_{n\to\infty} x_n = \frac{1}{2}$ (b) $\lim_{n\to\infty} x_n = \frac{2}{3}$

(c) $\lim_{n\to\infty} x_n = 1$ (d) $\{x_n\}$ is cauchy

Sol^n:- How to **THINK**

Recall some basic facts:-

- We say $\{x_n\}$ is cauchy in (X, d) if $d(x_n, x_m) \to 0$ as $n, m \to \infty$

- Here $x_n = \dfrac{x_{n-1} + x_{n-2}}{2} \Rightarrow 2x_n = x_{n-1} + x_{n-2}$

$\Rightarrow 2(x_n - x_{n-1}) = -(x_{n-1} - x_{n-2})$

$\Rightarrow x_n - x_{n-1} = \left(-\frac{1}{2}\right)(x_{n-1} - x_{n-2}) = \cdots = \left(-\frac{1}{2}\right)^{n-2}(x_2 - x_1)$

$\Rightarrow x_n - x_{n-1} = \left(-\frac{1}{2}\right)^{n-1}$ & $\sum_{n=1}^{\infty} \frac{1}{2^n} < \infty$

- Thus $\{x_n\}$ is cauchy in $\mathbb{R}$ (How??)

- Let $\lim_{n\to\infty} x_n = l$; $\because x_n = \dfrac{x_{n-1} + x_{n-2}}{2}$

$\Rightarrow 2x_n + x_{n-1} = 2x_{n-1} + x_{n-2} = \cdots = 2x_2 - x_1 = 2$

$\Rightarrow \lim_{n\to\infty} [2x_n + x_{n-1}] = 2 \Rightarrow 2l + l = 2 \Rightarrow \boxed{l = \frac{2}{3}}$

for NET-December. Batch Starts from 2nd July

Weekend 3rd June | 15th July.

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