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Today's IIScian Approach

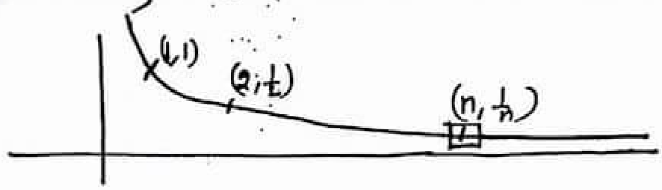
Let $X = \mathbb{N} \times \mathbb{Q}$ with subspace topology of the usual topology on $\mathbb{R}^2$ and $P = \{(n, \frac{1}{n}) : n \in \mathbb{N}\}$. Then

(a) P is open & closed in X (b) $\partial P \neq \phi$.

Sol:- How to $\boxed{\text{THINK}}$

First Recall some basic facts:-

- If $(X, \mathcal{T})$ is a top. space and $Y \subseteq X$ then subspace topology on Y is given by $\{Y \cap U : U \in \mathcal{T}\}$
- Here P is



- check P has no limit pts in X. so closed. (Justify).
- let $B_r(n, \frac{1}{n})$ be any nbhd of $(n, \frac{1}{n})$ then $\exists\ (n, q) \in B_r(n, \frac{1}{n})$ for $q \neq \frac{1}{n}$ and $q \in \mathbb{Q}$.

Hence $P^c \cap B_r(n, \frac{1}{n}) \neq \phi\ \forall\ r > 0$.

So, P is NOT open.

Thus $\partial P = \overline{P} \cap \overline{P^c} = P \cap X$ (How, $\overline{P^c} = X$) $= P$

Then $\partial P = P \neq \phi$.