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Linear Algebra

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Today's IIScian Approach

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Q. Let $V := \{ a_2 x^2 + a_1 x + a_0 : a_i \in \mathbb{R} \}$ be a Vector space over $\mathbb{R}$, $f : V \to \mathbb{R}$ be a linear functional s.t. $f(1+x) = 0$, $f(1-x^2) = 0$ and $f(x^2 - x) = 2$. Then $f(1+x+x^2) = \ldots ???$

Solution :-

How to $\boxed{\text{THINK}}$

Recall some basic Facts:

- $f : V(\mathbb{F}) \to U(\mathbb{F})$ is a Linear Transfor and $f(u_1) = v_1$, $T(u_2) = v_2$, $T(u_3) = v_3$

  Then $f(\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3) = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$

  [by Def^n of L.T.]

- let $1+x+x^2 = \alpha_1 (1+x) + \alpha_2 (1-x^2) + \alpha_3 (x^2 - x)$

  then $1+x+x^2 = \frac{3}{2}(1+x) - \frac{1}{2}(1-x^2) + \frac{1}{2}(x^2-x)$

  [Prove this !!!]

- Since $f : V \to \mathbb{R}$ is a linear functional

  so, $f(1+x+x^2) = \frac{3}{2} f(1+x) - \frac{1}{2} f(1-x^2) + \frac{1}{2} f(x^2-x)$

- i.e. $f(1+x+x^2) = \frac{3}{2} \times 0 - \frac{1}{2} \times 0 + \frac{1}{2} \times 2 = 1.$

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