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Today's IIScian Approach

If $R := \dfrac{\mathbb{Q}[x]}{\langle x^4-1\rangle}$, then the number of maximal ideals of R is

(a) 1 (b) 2 (c) 3 ~~(d)~~ 4.

Sol^n:- How to $\boxed{\text{THINK}}$

First recall some basic facts:-

- If R is CRU and I is an ideal of R then I is maximal $\Leftrightarrow \frac{R}{I}$ is field.
- If I, J are ideals of R s.t. $J \subseteq I \subseteq R$ then $\frac{R}{I} \approx \frac{R/J}{I/J}$. (Justify ...).
- $x^4-1 = (x^2-1)\cdot(x^2+1) = (x-1)(x+1)(x^2+1)$.
- $\langle p(x)\rangle$ is maximal in $\mathbb{Q}[x] \Leftrightarrow p(x)$ is irr. over $\mathbb{Q}$
- Here $\langle x-1\rangle$, $\langle x+1\rangle$ & $\langle x^2+1\rangle$ are maximal in $\mathbb{Q}[x]$.
- Hence $\dfrac{\mathbb{Q}[x]}{\langle x-1\rangle} \approx \dfrac{\mathbb{Q}[x]/\langle x^4-1\rangle}{\langle x-1\rangle/\langle x^4-1\rangle}$ is field

  $\Rightarrow \dfrac{\langle x-1\rangle}{\langle x^4-1\rangle}$ is maximal ideal in R.
- Similarly $\dfrac{\langle x+1\rangle}{\langle x^4-1\rangle}$ and $\dfrac{\langle x^2+1\rangle}{\langle x^4-1\rangle}$ are maximal ideals in R
- These are ALL Maximal ideals of R.

  Hence Ans:- $\boxed{C}$ ▨.

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