NOBLE FORUM
a forum of IISc / IMSc graduates...
Centre for Excellence in MATHEMATICS
NBHM/NET/GATE/IIT-JAM/TIFR/ISI/CMI

NET June 2017.

Sumit Kumar
IISc.-Bangalore (alumnus)
IMSc.-Chennai (alumnus)

Today's IIScian Approach

Let $f: \mathbb{R} \to \mathbb{R}$ be a cts function s.t. $f(x) \geq 0$ and $f(x) = 0$ if $|x| \geq 1$. Also $\int_{-\infty}^{\infty} f(t)\,dt = 1$.

Let $f: \mathbb{R} \to \mathbb{R}$ be a cts function. then

Evaluate $\lim_{\epsilon \to 0} \frac{1}{\epsilon} \int_{-\infty}^{\infty} f\left(\frac{x}{\epsilon}\right) f(x)\,dx$.

Sol^n:- How to THINK

First recall some basic facts:-

- If $f_n \to f$ unif on $[-1,1]$ and f_n's are cts then $\int_{-1}^{1} f_n(x)\,dx \to \int_{-1}^{1} f(x)\,dx$ (Justify)
- $\frac{1}{\epsilon} \int_{-\infty}^{\infty} f\left(\frac{x}{\epsilon}\right) f(x)\,dx = \int_{-1}^{1} f(t)\, f(t\epsilon)\,dt$; (put $\frac{x}{\epsilon} = t$)
- Define $g_n(t) = f(t)\, f\left(\frac{t}{n}\right)$, $g(t) = f(t)\, f(0)$
- check $g_n \to g$ unif on $[-1,1]$ and g_n's cts.
- Thus $\lim_{n \to \infty} \int_{-1}^{1} g_n(t)\,dt = \int_{-1}^{1} g(t)\,dt = f(0)$.
- i.e. $\lim_{\epsilon \to 0} \frac{1}{\epsilon} \int_{-\infty}^{\infty} f\left(\frac{x}{\epsilon}\right) f(x)\,dx = f(0)$. ▨

THINK Like a child!

NBHM / NET December ...Regular Batch... 2nd July...