

Today's || Scian Approach

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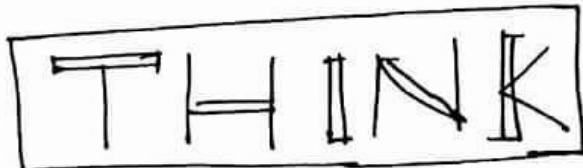
If $T : \ell^\infty \rightarrow \ell^2$ given by
 $T(a_1, a_2, \dots) := (a_1, \frac{a_2}{2}, \frac{a_3}{3}, \dots)$

Then wOTF w/are true

- (a) T is cts
- (b) T maps ℓ^∞ onto ℓ^2
- (c) T^{-1} exists and cts
- (d) T is unif. cts.

Sol:-

How to



- Recall some basic facts:-
- If X and Y are normed v.s. and $T: X \rightarrow Y$ is a L.T. then T is cts if $\exists M > 0$ s.t.
 $\|Tx\|_Y \leq M \|x\|_X ; \forall x \in X.$
- Here $X = \ell^\infty, Y = \ell^2, T(a_1, \dots) = (a_1, \frac{a_2}{2}, \dots)$
- $\|T(a_1, a_2, \dots)\|_2 = \left(\sum | \frac{a_n}{n} |^2 \right)^{\frac{1}{2}} \leq \|a\|_\infty \left(\sum \frac{1}{n^2} \right)^{\frac{1}{2}}$
[Justify].
- Thus $\exists M = \frac{\pi}{\sqrt{6}}$ s.t. $\|Tx\|_2 \leq \frac{\pi}{\sqrt{6}} \|x\|_\infty$.
- Hence T is uniform cts. (How???)
- T is NOT ONTO: $\because \left\{ \frac{1}{n^{\frac{1}{4}}} \right\}_{n=1}^\infty \in \ell^2$ and
 $\frac{a_n}{n} = \frac{1}{n^{\frac{5}{4}}} \Leftrightarrow a_n = n^{\frac{1}{4}}$ but $\left\{ \frac{1}{n^{\frac{1}{4}}} \right\}_{n=1}^\infty \notin \ell^\infty$.
- Hence T^{-1} does NOT make sense. Also if possible

then $T^{-1}(a_1, \dots) = (a_1, 2a_2, \dots, na_n, \dots)$

and as $a_n = \frac{1}{n^{\frac{1}{4}}} : \{a_n\} \in \ell^2$ but $T^{-1}(a_1, \dots) = \left(1, 2^{\frac{1}{4}}, \dots, n^{\frac{1}{4}}, \dots \right)$

So, go through basic concepts of functional analysis.
feel free to ask your doubts.

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