Today's IIScian Approach

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If $T : \ell^\infty \to \ell^2 (\ell^2)$ given by

$$T(a_1, a_2, \dots) := \left(a_1, \frac{a_2}{2}, \frac{a_3}{3}, \dots\right)$$

Then WOTF is/are true

(a) T is cts (b) T maps ℓ^∞ <u>ONTO</u> ℓ^2

(c) T^{-1} exists and cts (d) T is Unif. cts.

<u>Solⁿ</u> :- How to $\boxed{\text{THINK}}$

- Recall some <u>basic facts</u> :-
- If X and Y are normed v.s. and $T : X \to Y$ is a L.T. then T is cts if $\exists$ $M > 0$ s.t.

$$\|Tx\|_Y \le M \|x\|_X \; ; \forall x \in X.$$

- Here $X = \ell^\infty$, $Y = \ell^2$, $T(a_1, \dots) = \left(a_1, \frac{a_2}{2}, \dots\right)$
- $\|T(a_1, a_2, \dots)\|_2 = \left(\sum \left|\frac{a_n}{n}\right|^2\right)^{\frac{1}{2}} \le \|a\|_\infty \left(\sum \frac{1}{n^2}\right)^{\frac{1}{2}}$ [Justify].
- Thus $\exists$ $M = \frac{\pi}{\sqrt{6}}$ s.t. $\|Ta\|_2 \le \frac{\pi}{\sqrt{6}} \|a\|_\infty$.
- Hence T is Uniform cts. (How???)
- T is <u>NOT</u> <u>ONTO</u> : $\because \left\{\frac{1}{n^{\frac{3}{4}}}\right\}_{n\ge1}^\infty \in \ell^2$ and

$$\frac{a_n}{n} = \frac{1}{n^{\frac{3}{4}}} \iff a_n = n^{\frac{1}{4}} \text{ but } \left\{n^{\frac{1}{4}}\right\}_{n\ge1}^\infty \notin \ell^\infty.$$

- Hence T^{-1} does <u>NOT</u> make sense. Also if possible then $T^{-1}(a_1, \dots) = (a_1, 2a_2, \dots, na_n, \dots)$ and as $a_n = \frac{1}{n^{\frac{3}{4}}}$; $\{a_n\} \in \ell^2$ but $T^{-1}(a_1, \dots) = (1, 2^{\frac{1}{4}}, \dots, n^{\frac{1}{4}}, \dots) \notin \ell^\infty$.

- So, Go through basic concepts of Functional Analysis.
- Feel free to ask your doubts.

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