

Today's IIScian Approach

Q. let $A_{4 \times 4}$ matrix. Suppose that the null space $N(A)$ of A is

$N(A) := \{ (x, y, z, w) \in \mathbb{R}^4 : x+y+z=0, x+y+w=0 \}$. Then

① $\dim(\text{column space}(A)) = 1$ ③ $\text{rank}(A) = 1$

② $\dim(\text{column space}(A)) = 2$

④ $S = \{ (1, 1, 0), (1, 1, 0, 1) \}$ is a basis of $N(A)$.

Sol:-

How to T H I N K

First recall some basic facts:-

- Column space of a matrix is the image of the corresponding matrix transformation.
- The dimension of the column space is called the rank of the matrix.

• Here $(x, y, z, w) \in N(A) \Leftrightarrow x+y+z=0 = x+y+w$
 i.e. $z = -x-y, w = -x-y$.

• $N(A) = \{ (x, y, -x-y, -x-y) : x, y \in \mathbb{R} \}$.

• $\text{nullity}(A) = \dim(N(A)) = 2$

• By Rank-nullity Th:- $\text{rank}(A) + \text{nullity}(A) = 4$
 ($\because A_{4 \times 4}$)

i.e. $\text{rank}(A) + 2 = 4 \Rightarrow \boxed{\text{rank}(A) = 2}$ □

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