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Today's IIScian Approach

Q Let $f(x) = \tan^{-1}x$; $x \in \mathbb{R}$, then

1. $\exists$ a polynomial $p(x)$ satisfying $p(x) f'(x) = 1$ $\forall x \in \mathbb{R}$
2. $f^n(0) = 0$ $\forall$ even +ve Integer
3. The sequence $\{f^n(0)\}$ is unbounded
4. $f^n(0) = 0$ $\forall n$.

Sol:- How to $\boxed{\text{THINK}}$

First recall some basic facts:-

. The power series of $\tan^{-1}x$ about $x=0$ is

$$\tan^{-1}x := x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$$

. Thus $\dfrac{f^n(0)}{n!} = \begin{cases} 0 & : n \text{ is even} \\ \dfrac{(-1)^k}{n} & ; n = 2k+1 \end{cases}$

. i.e. $f^n(0) = \begin{cases} 0 & : n \text{ is even} \\ (-1)^k (n-1)! & : n = 2k+1 \end{cases}$

. Thus $f^n(0) = 0$ if n is even, otherwise $f^n(0) = (-1)^{\frac{n-1}{2}} (n-1)!$

. Hence $\{f^n(0)\}$ is unbounded. (✓)

. Also $f'(x) = \dfrac{1}{1+x^2} \Rightarrow (1+x^2) \cdot f'(x) = 1$

$\Rightarrow p(x) f'(x) = 1$ $\forall x \in \mathbb{R}$.

Ans:- ①, ② & ③.