CSIR NET JUNE 17 : A; 68 B; 63 C; 62.

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Today's IIScian Approach

Q suppose $f : \mathbb{R}^n \to \mathbb{R}$ is given by

$$f(\underline{x}) = a_1 x_1^2 + a_2 x_2^2 + \cdots + a_n x_n^2, \quad \underline{x} = (x_1, \ldots, x_n)$$

atleast one $a_j \neq 0$. Then we can conclude that

1. f is NOT everywhere differentiable
2. $(\nabla f)(\underline{x}) \neq 0 \ \forall \ \underline{x} \in \mathbb{R}^n$
3. If $\nabla f(\underline{x}) = 0$ Then $f(\underline{x}) = 0$
4. If $\underline{x} \in \mathbb{R}^n$ s.t. $f(\underline{x}) = 0$ then $(\nabla f)(\underline{x}) = 0$.

Sol :- How to $\boxed{\text{THINK}}$

· First recall some basic facts :-

· $(\nabla f)(\underline{x}) = \frac{\partial f}{\partial x_1} e_1 + \frac{\partial f}{\partial x_2} e_2 + \cdots + \frac{\partial f}{\partial x_n} e_n$

· Here f is a polynomial & hence differentiable.

· Here $(\nabla f)(\underline{x}) = 2a_1 x_1 e_1 + 2a_2 x_2 e_2 + \cdots + 2a_n x_n e_n$

$$= 2(a_1 x_1, a_2 x_2, \ldots, a_n x_n)$$

· Take $\underline{x} = \vec{0} = (0, \ldots, 0)$ Then $(\nabla f)(\underline{x}) = 0$.

So ② is false

3 If $\nabla f(\underline{x}) = 0 \Leftrightarrow a_i x_i = 0 \ ; \ 1 \leq i \leq n$

$$\Rightarrow a_i x_i^2 = 0 \cdot x_i = 0 \ ; \ 1 \leq i \leq n$$

· $\Rightarrow$ so, $f(\underline{x}) = a_1 x_1^2 + \cdots + a_n x_n^2 = 0$, Here ③ is true

· Take $n = 2$, $f(x_1, x_2) = x_1^2 - x_2^2$; $f(1,-1) = 0$

But $\nabla f = (2x_1, -2x_2)$, $(\nabla f)(1,-1) = (2, 2) \neq 0$] so ④ is false

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