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Q. Which of the following polynomials are irreducible in $\mathbb{Z}[x]$

(1) $x^4+10x+5$ (2) x^3-2x+1

(3) x^4+x^2+1 (4) x^3+x+1.

Sol:- How to THINK

First recall some basic facts:-

- a non constant poly $f(x)$ is said to be irreducible over a field $\mathbb{F}$ if its coeff. belongs to $\mathbb{F}$ and it cannot be factored into the product of two non-constant polynomials with coefficients in $\mathbb{F}$.
- If $f(x) \in \mathbb{Z}[x]$ and primitive then $f(x)$ is irreducible in $\mathbb{Z}[x] \Leftrightarrow f(x)$ is irr. in $\mathbb{Q}[x]$.
- By Eisenstein's criterion: $f(x) = x^4+10x+5$ is irr. over $\mathbb{Q}$, and $f(x)$ is monic so over $\mathbb{Z}$.

Primitive polynomial

- $f(x)=x^3-2x+1$; $x=1$ is a root of $f(x)$, so red.
- $f(x)=x^4+x^2+1 = (x^2-x+1)(x^2+x+1)$, so red.
- Take $p=2$ and apply mod p Test, $f(x)=x^3+x+1$ is irr. over $\mathbb{Z}_2 \Rightarrow$ over $\mathbb{Q} \Rightarrow$ over $\mathbb{Z}$.