

Today's IIScian Approach

Q) Which of the following polynomials are irreducible in  $\mathbb{Z}[x]$

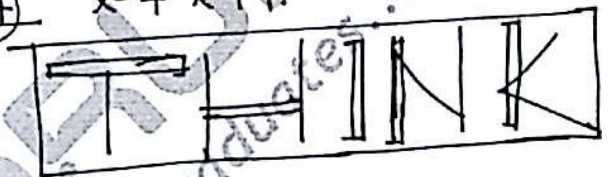
①  $x^4 + 10x + 5$

②  $x^3 - 2x + 1$

③  $x^4 + x^2 + 1$

④  $x^3 + x + 1$

Sol<sup>n</sup> :- How to



First recall some basic facts:-

• a non constant poly  $f(x)$  is said to be irreducible over ~~over~~ a field  $\mathbb{F}$  if its coeff. belongs to  $\mathbb{F}$  and it cannot be factored into the product of two non-constant polynomials with coefficients in  $\mathbb{F}$ .

• If  $f(x) \in \mathbb{Z}[x]$  and primitive then  $f(x)$  is irreducible in  $\mathbb{Z}[x] \Leftrightarrow f(x)$  is irr. in  $\mathbb{Q}[x]$ .

• By Eisenstein's criterion:  $f(x) = x^4 + 10x + 5$  is irr. over  $\mathbb{Q}$ , and  $f(x)$  is monic so over  $\mathbb{Z}$ .

•  $f(x) = x^3 - 2x + 1$ ;  $x = 1$  is a root of  $f(x)$ , so irr. red.   
 $f(x) = x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$ , so red.   
 • Take  $p = 2$  and apply mod  $p$  Test,  $f(x) = x^3 + x + 1$  is irr. over  $\mathbb{Z}_2 \Rightarrow$  over  $\mathbb{Q} \Rightarrow$  over  $\mathbb{Z}$ .

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