NOBLE FORUM
a forum of IISc / IMSc graduates...
Centre for Excellence in MATHEMATICS
NBHM/NET/GATE/IIT-JAM/TIFR/ISI/CMI

Sumit Kumar
IISc.-Bangalore (alumnus)
IMSc.-Chennai (alumnus)

Today's IIScian Approach

Q for $n \geq 1$, $f_n(x) = x e^{-nx^2}$, $x \in \mathbb{R}$, Then the seq $\{f_n\}$ is

1. uniformly convergent on $\mathbb{R}$
2. uniformly convergent only on compact subsets of $\mathbb{R}$
3. bounded and NOT uniformly convergent on $\mathbb{R}$
4. a seq. of unbounded functions.

Sol^n:- How to $\boxed{\text{THINK}}$

First recall some basic facts:-

- We say $f_n \to f$ uniformly on $\mathbb{R}$ if for given $\epsilon > 0$, $\exists$ $N \in \mathbb{N}$ s.t. $\sup_{x \in \mathbb{R}} |f_n(x) - f(x)| < \epsilon$ $\forall n \geq N$.
- Here $f_n(x) = x e^{-nx^2}$, & $f(x) = \lim_{n \to \infty} f_n(x) = 0$.
- To evaluate $\sup_{x \in \mathbb{R}} |f_n(x) - f(x)| = \sup_{x \in \mathbb{R}} |x e^{-nx^2}|$
- solve, $\frac{d}{dx}\left(x e^{-nx^2}\right) = 0 \Rightarrow x^2 = \frac{1}{2n}$.
- So, $\sup_{x \in \mathbb{R}} |f_n(x) - f(x)| = \frac{1}{\sqrt{2n}} e^{-\frac{1}{2}} \to 0$ as $n \to \infty$.
- Thus $f_n \to f$ uniformly on $\mathbb{R}$ ▨

—— o ——

→ You have talent, I will definitely support you...