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## Today's IIScian Approach

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Let $\mathbb{C}^*$ be a group under multiplication. Then
(1) $\mathbb{C}^*$ is cyclic (2) Every finite subgroup of $\mathbb{C}^*$ is cyclic (3) $\mathbb{C}^*$ has finitely many finite subgps
(4) Every proper subgroups of $\mathbb{C}^*$ is cyclic.

Sol$^n$:- How to $\boxed{\text{THINK}}$.

Recall the basic facts:.

- An infinite group. $G$ is cyclic iff $G \simeq \mathbb{Z}$.
- Here $\mathbb{C}^*$ is Uncountable gp so $\mathbb{C}^* \not\simeq \mathbb{Z}$
- Let $H \trianglelefteq \mathbb{C}^*$ and $o(H)=n$, apply Langrange's therem $\forall z \in H$; $z^n = 1$, so, $H = \langle e^{\frac{2\pi i}{n}} \rangle$.
- for each $n \in \mathbb{N}$, $\exists H_n := \langle e^{\frac{2\pi i}{n}} \rangle$ and $H_n \trianglelefteq \mathbb{C}^*$, $\forall n \neq m$, $H_n \neq H_m$, so $\mathbb{C}^*$ has infinitely many finite subgps.
- Take $H = \mathbb{R}^* \trianglelefteq \mathbb{C}^*$ and $\mathbb{R}^*$ is Uncountable so, $\mathbb{R}^*$ is NOT cyclic.

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