· Mathematics is NOT about Problem Solving – It's about Understanding...
→ Regular Batch... NET June 2018... 5th JAN... Limited seats...
...Crash Course... NET Dec 2017... started.

Today's IIScian Approach

consider the topology $\tau = \{ U \subseteq \mathbb{Z} : U^c \text{ is finite or } 0 \notin U \}$ on $\mathbb{Z}$. Then the topological space $(\mathbb{Z}, \tau)$ is

(a) cpt but NOT connected (b) Connected but NOT cpt

(c) Both compact and connected (d) Neither cpt Nor Connected

Sol:- How to THINK

First recall some basic facts:-

- space X is said to be cpt if every open cover of X has a finite subcover.
- let $\{G_\alpha\}_{\alpha \in I}$ be an open cover of $\mathbb{Z}$ ie. $\mathbb{Z} = \bigcup_{\alpha \in I} G_\alpha$.
  - If some G_α^c is finite then we are done !!! [How???]
  - otherwise $0 \notin G_\alpha \ \forall \alpha \Rightarrow 0 \notin \bigcup_{\alpha \in I} G_\alpha = \mathbb{Z}$ $\rightarrow\leftarrow$
  - Thus $\mathbb{Z}$ is cpt top space.
- space X is said to be connected if ONLY subsets of X which are both open and closed are X and ϕ.
- Here take $G = \mathbb{Z} \setminus \{1\}$,
  - $G^c = \{1\}$, finite so, G is open in $\mathbb{Z}$
  - $0 \notin \{1\} = G^c \Rightarrow G^c$ is is open $\Rightarrow G$ is closed
  - So G is Both open & closed, $G \neq \phi$, $G \neq \mathbb{Z}$.
  - Hence $(\mathbb{Z}, \tau)$ is NOT connected...